

**THE
PLATONIC SOLIDS
BOOK**

DAN RADIN

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for my daughter Chelsea

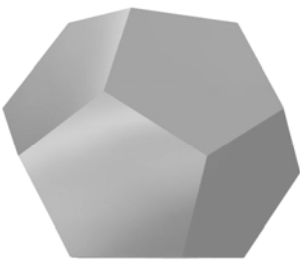
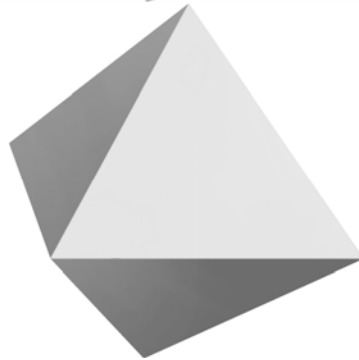
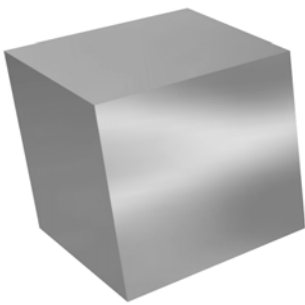






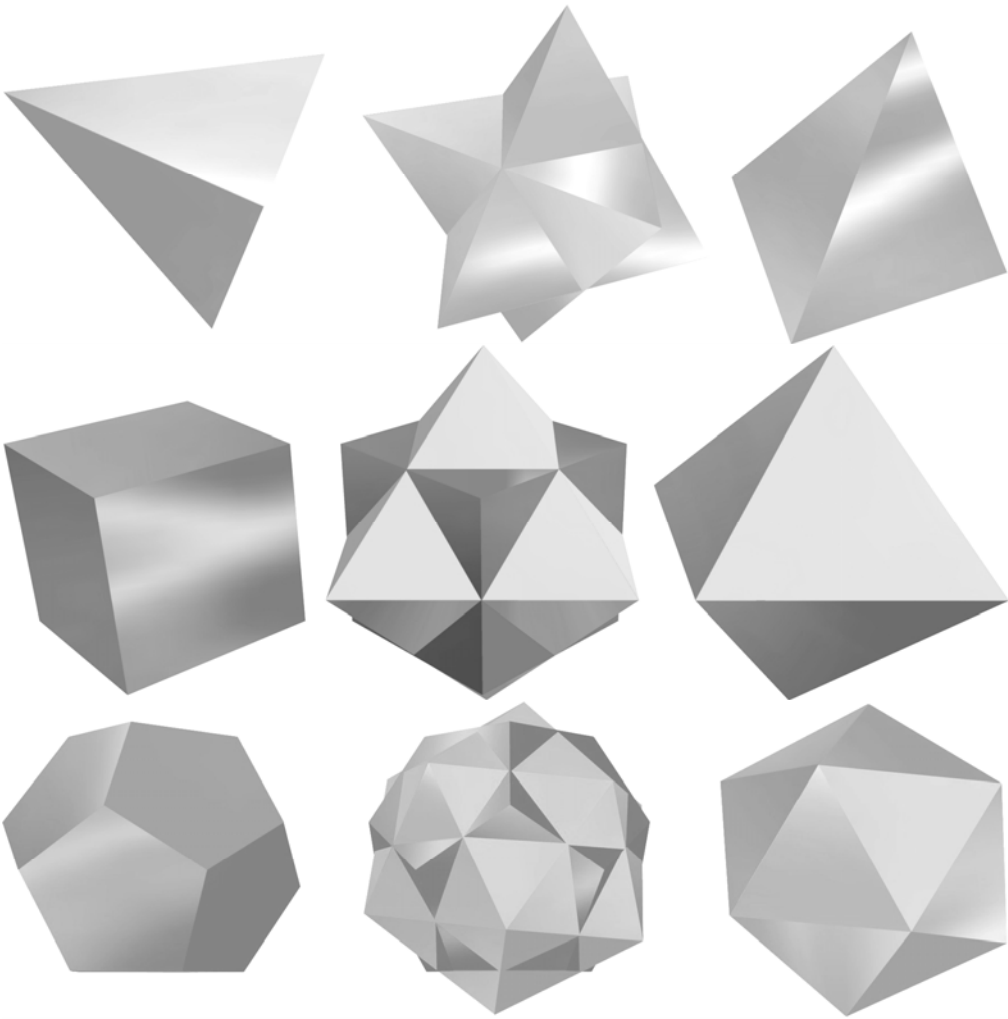


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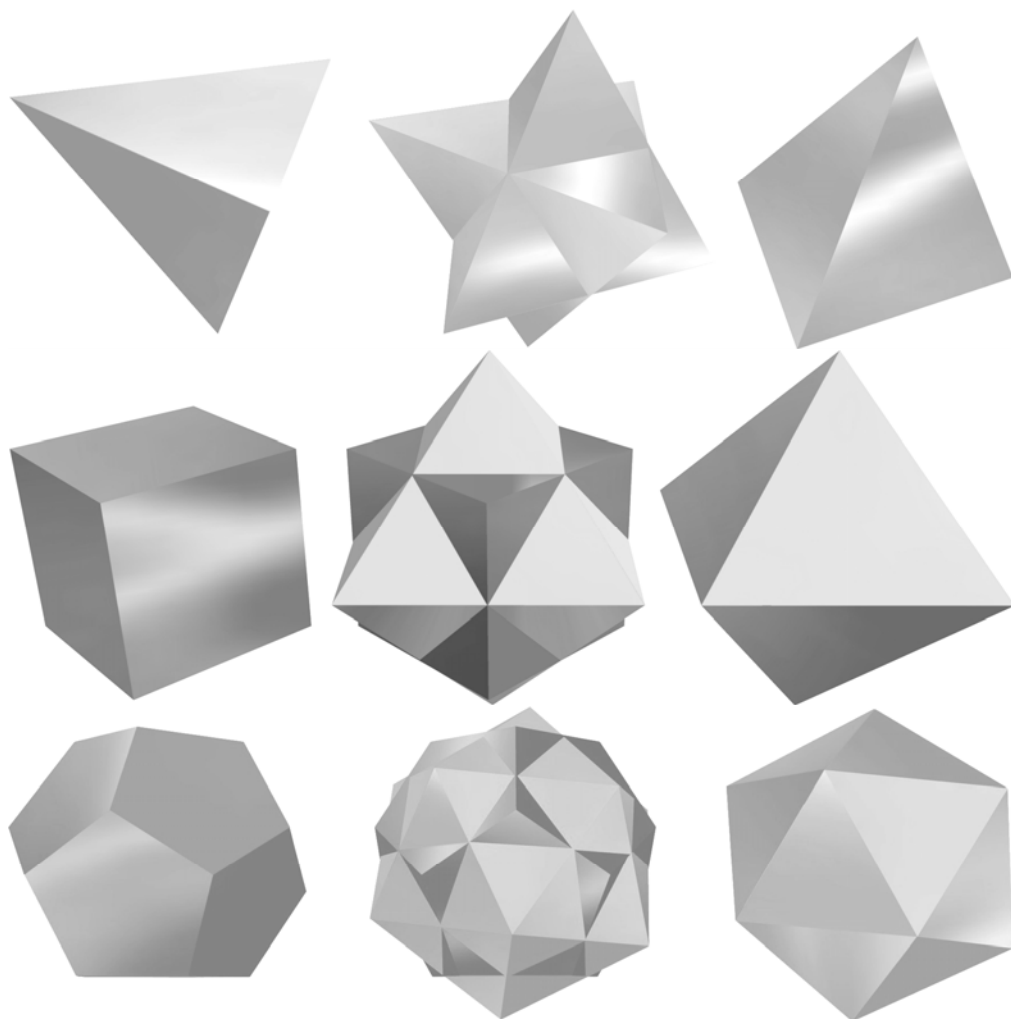


INTRODUCTION

The Platonic solids are the five most symmetric examples of polyhedra. The word, “polyhedra” is the plural form of the word, “polyhedron.” I once read that the direct translation of the Greek word, “polyhedron,” is “many seats.” Apparently, “hedron” means seat and a “cathedral” is a place where people sit. Today, mathematicians generally translate it as a solid having many flat faces. But I can see how seats could work too. Maybe you can sit on them in many ways, or they contain many surfaces upon which they can sit. By “most symmetric,” I mean that there are many ways that you can turn them around and have them still appear the same from different angles. I also mean that they look the same if you view them in a mirror.

The Platonic solids get their name from the Greek philosopher, Plato, who wrote about them. They were, in fact, known long before Plato by many different cultures. Plato wrote about them in his book *Timaeus*. This work was a mixture of philosophy, science, mathematics, and theology, which is not surprising since, at that time, the four fields were all considered part of the same whole. In *Timaeus*, Plato came up with an early version of atomic chemistry where all of matter was made up of combinations of these five shapes at a microscopic scale, and where it was these shapes that gave matter its properties.

It is not surprising that Plato believed that the gods had chosen these five most perfect forms from which to make all others. I know that I find them somehow intrinsically attractive, fascinating, and magical. I hope that you too will appreciate them as you read through this book and look at all of the beautiful images.



CHAPTER 1

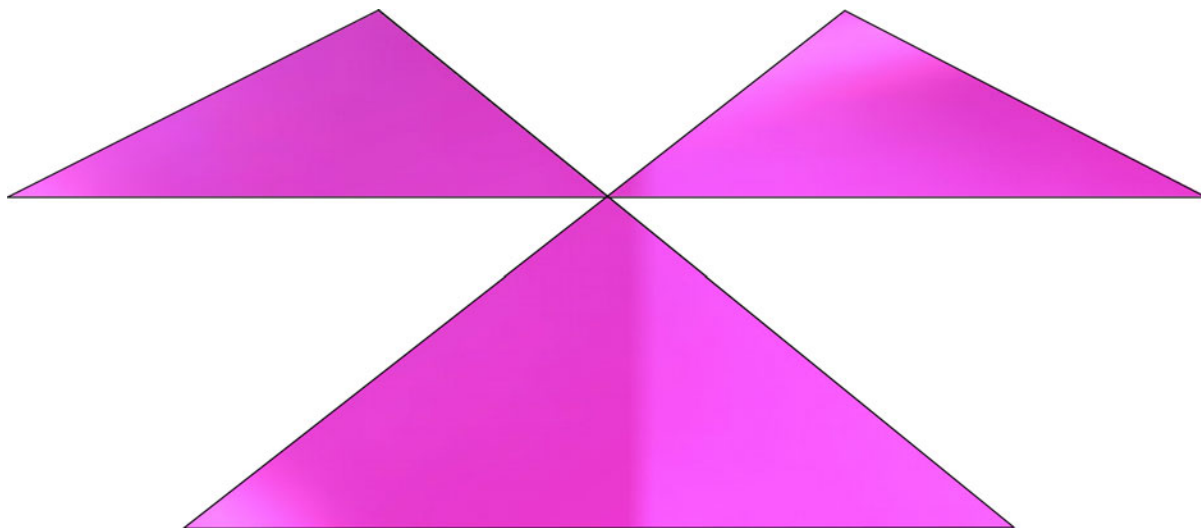
HOW TO MAKE A PLATONIC SOLID

All five Platonic solids are made from three different regular polygons: the equilateral triangle, the square, and the regular pentagon. To be a Platonic solid, all of the polygon faces must be identical and the same number of faces must meet together at each vertex. “Vertex” is the word mathematicians use for the corners or points. The plural of “vertex” is “vertices.”

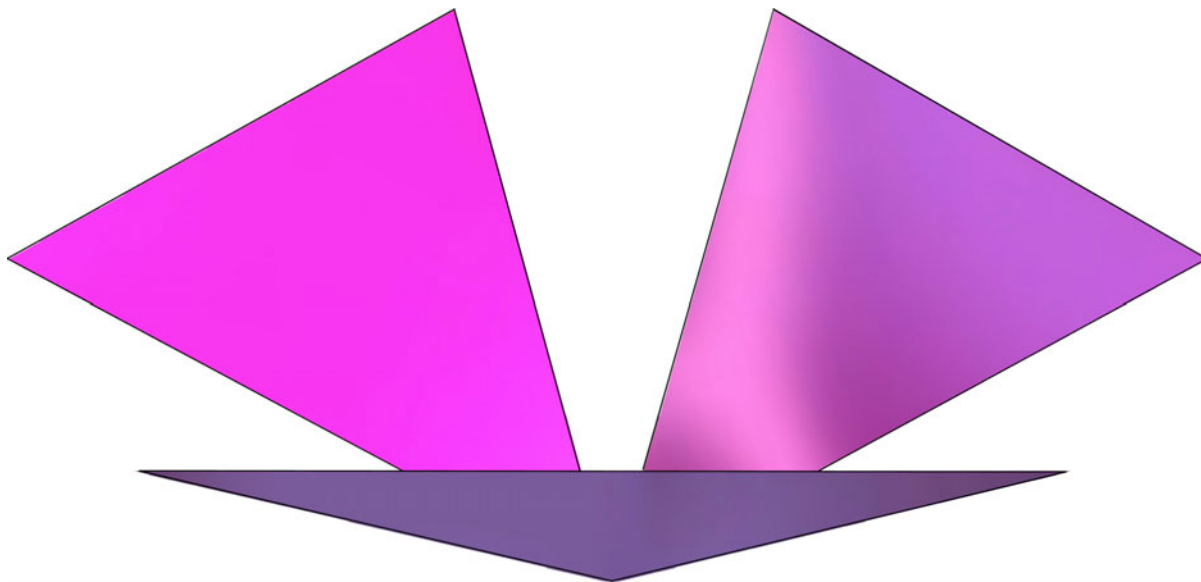
The remainder of this chapter is devoted to instructions for making each of the five Platonic solids. After you finish reading it, you may want to skip to chapter three and start building your own models. Or, you may decide to just read on and enjoy the pretty pictures.

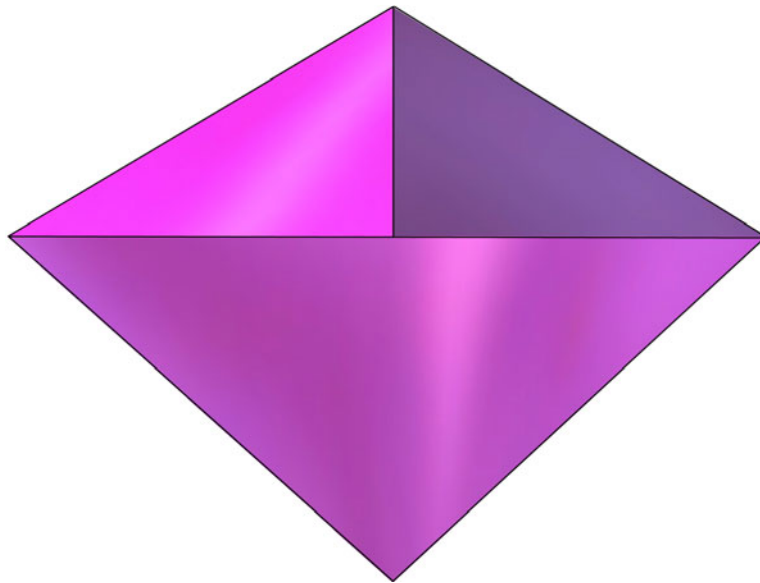
The Tetrahedron

The most basic Platonic solid is called the tetrahedron. The word “tetra” is Greek for four. The tetrahedron has four triangular faces. To make a tetrahedron, place three equilateral triangles point-to-point on a flat plane.

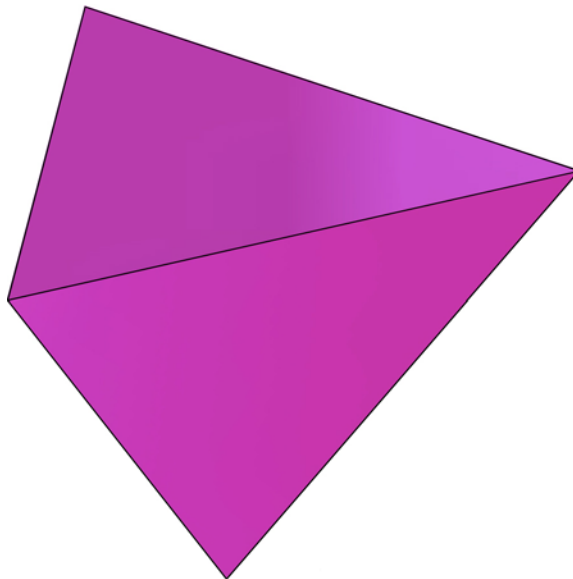


With the center points still on the plane, swing the triangles up out of the plane.





Place a fourth equilateral triangle on the top.

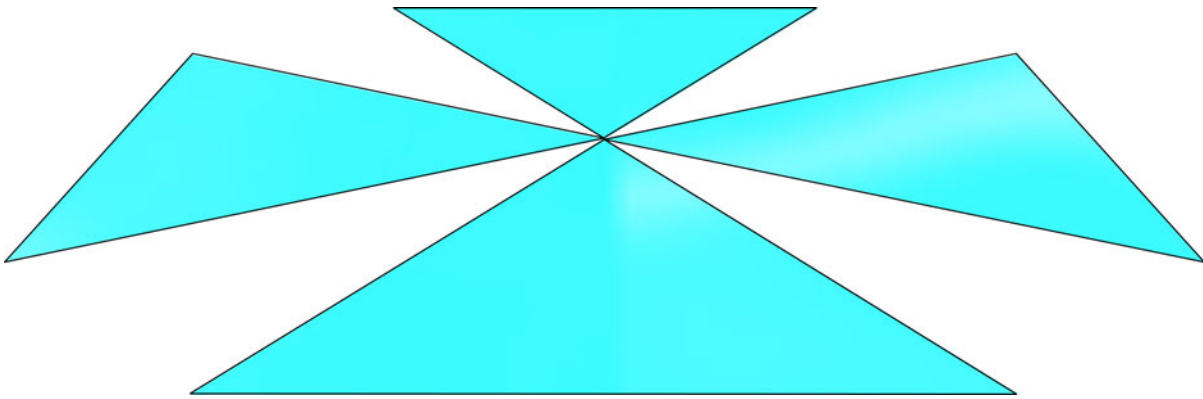


In a certain way, the tetrahedron is the most symmetric of the Platonic solids. Notice that each of its faces has three sides and that three faces meet at each vertex. The tetrahedron is the only Platonic solid with this dual nature between the number of

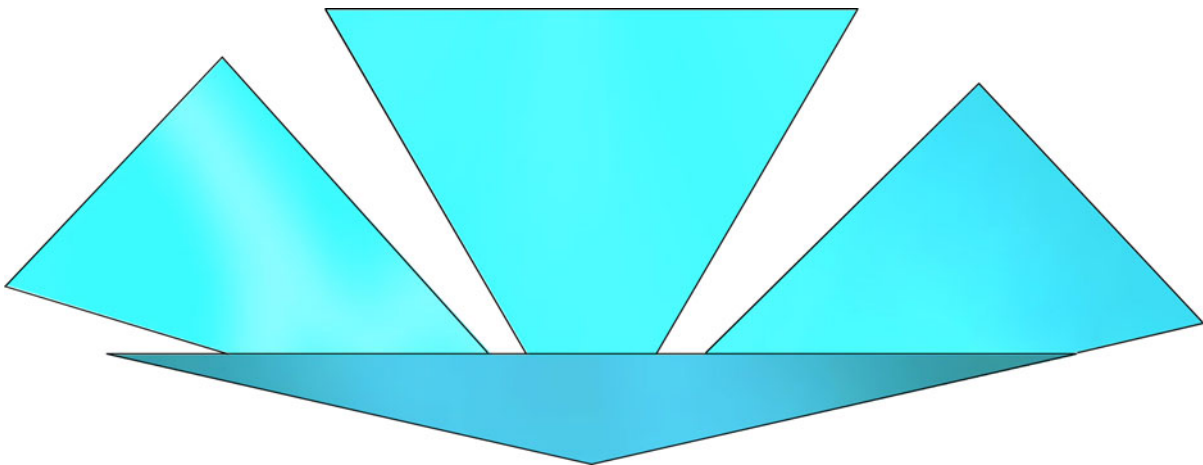
sides on each face and the number of faces meeting at each vertex. You will see, in chapter two, that this feature of the tetrahedron has a very special consequence.

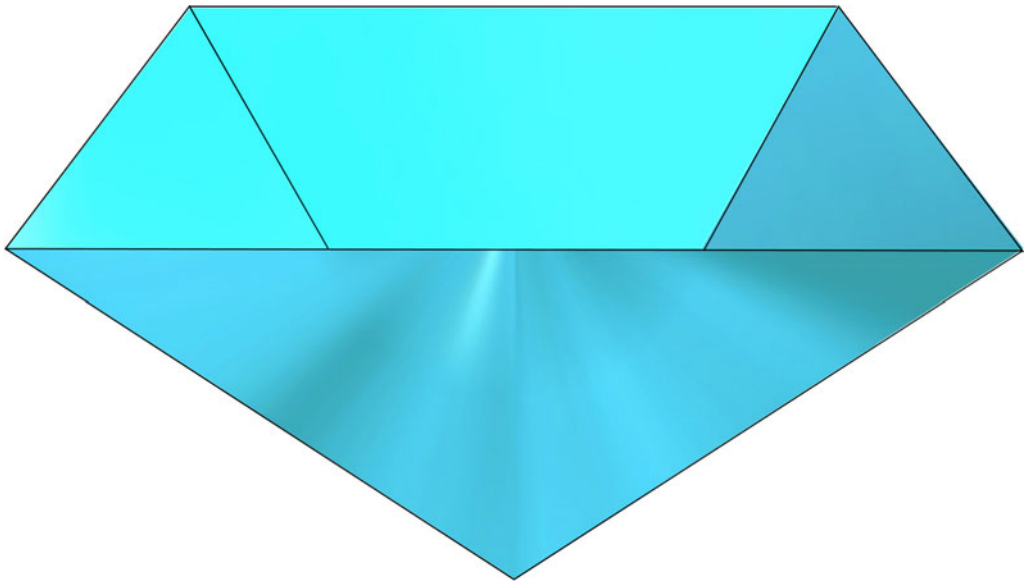
The Octahedron

The next triangle-based Platonic solid is called the octahedron. The word “octa” is Greek for eight. The octahedron consists of eight equilateral triangle faces. To make an octahedron, place four equilateral triangles point-to-point on a flat plane.

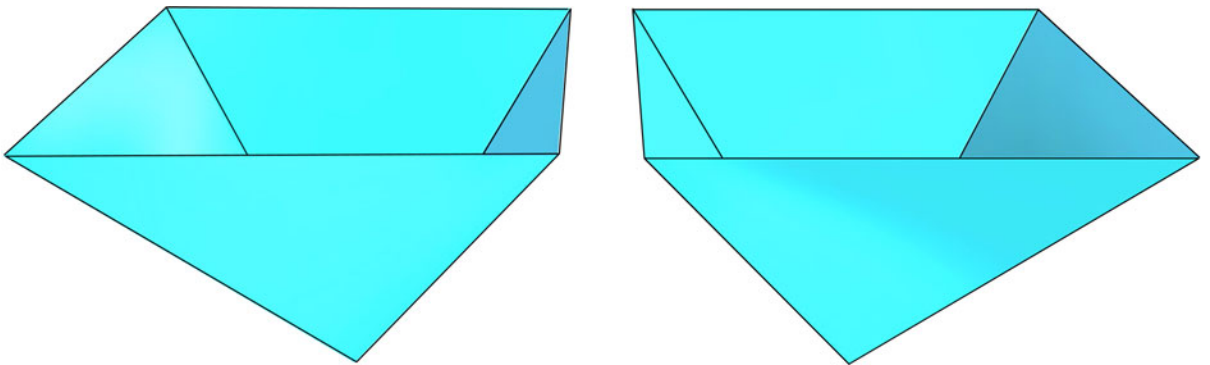


With the center points still on the plane, swing the triangles up out of the plane.

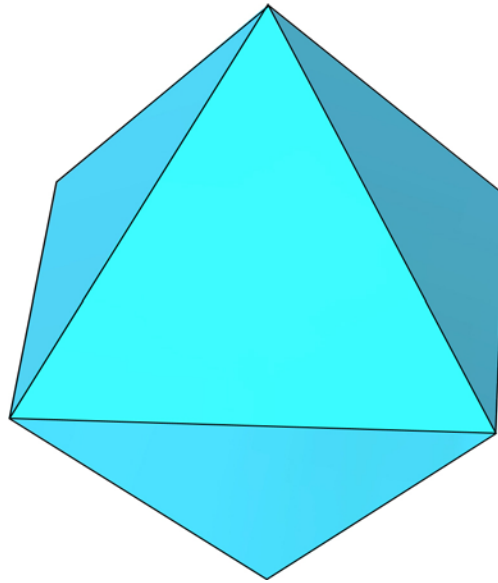




Start the whole process over again with four more triangles on the plane point-to-point. Swing the new triangles up out of the plane.



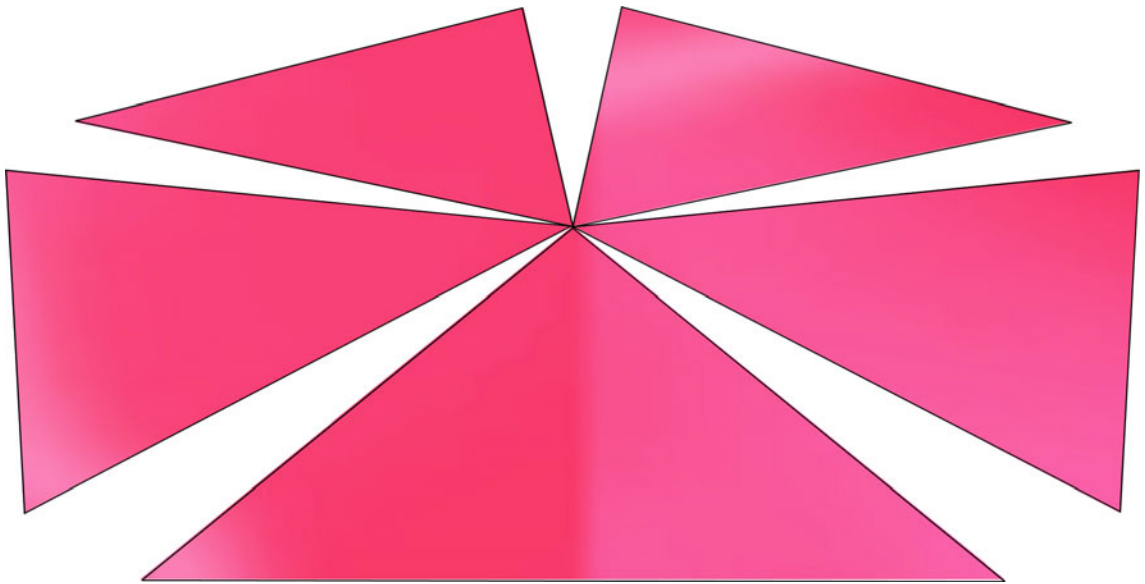
Place the second set of four triangles on top of the first set.



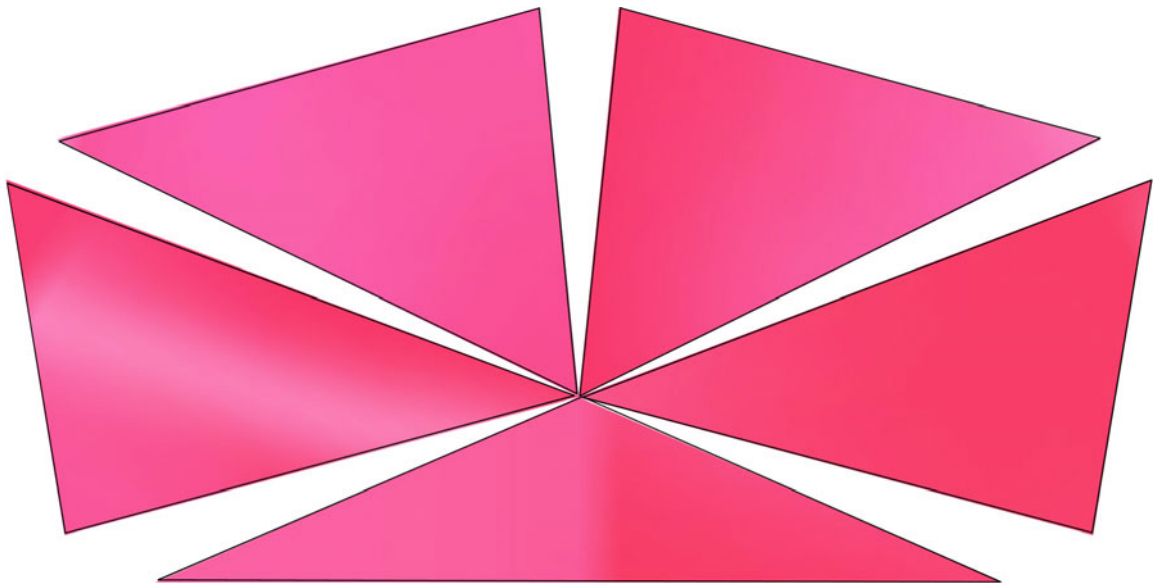
This is the octahedron, the second triangle-based Platonic solid.

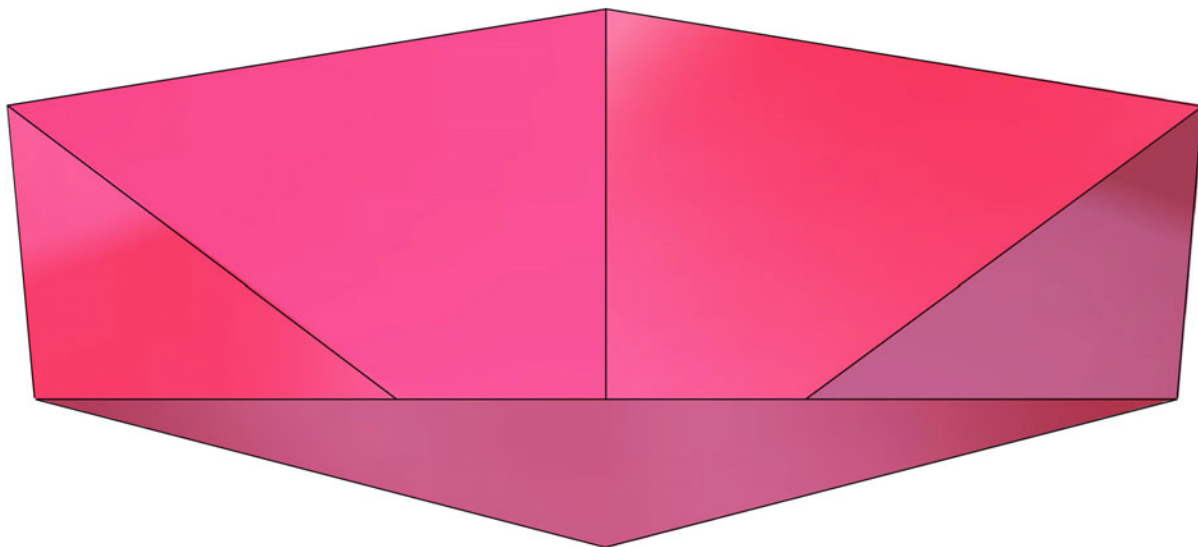
The Icosahedron

The last triangle-based Platonic solid is called the icosahedron. The word “icosa” is Greek for twenty. The icosahedron consists of twenty equilateral triangle faces. To make an icosahedron, place five equilateral triangles point-to-point on a flat plane.

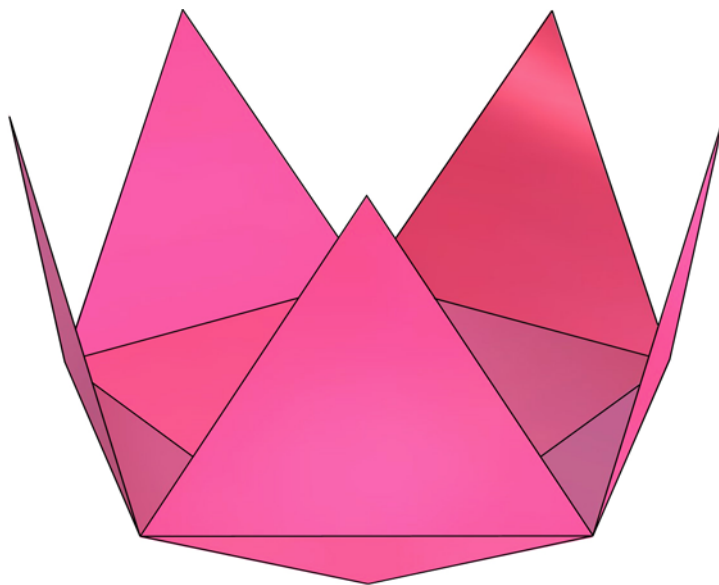


With the center points still on the plane, swing the triangles up out of the plane.

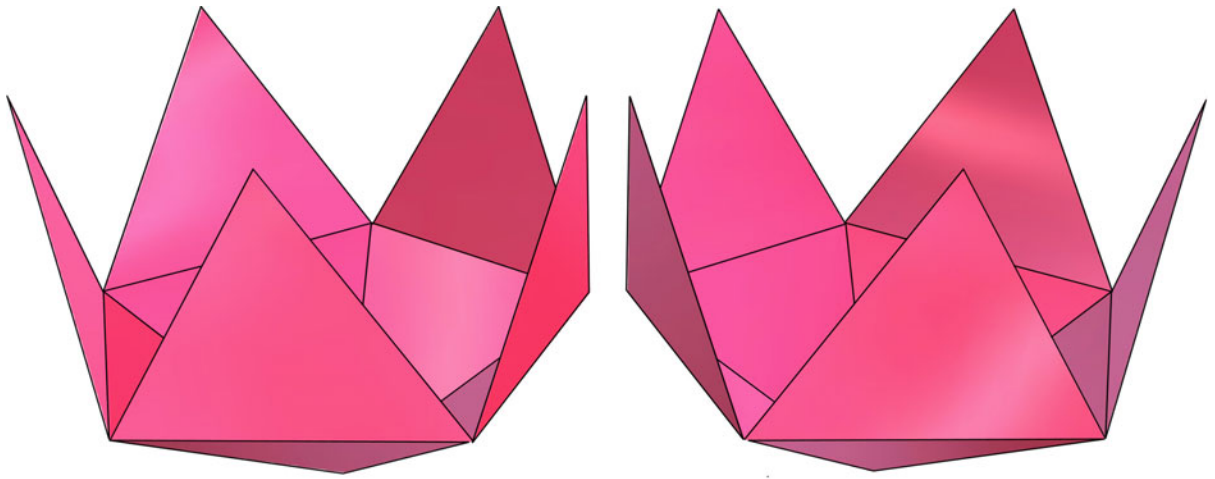




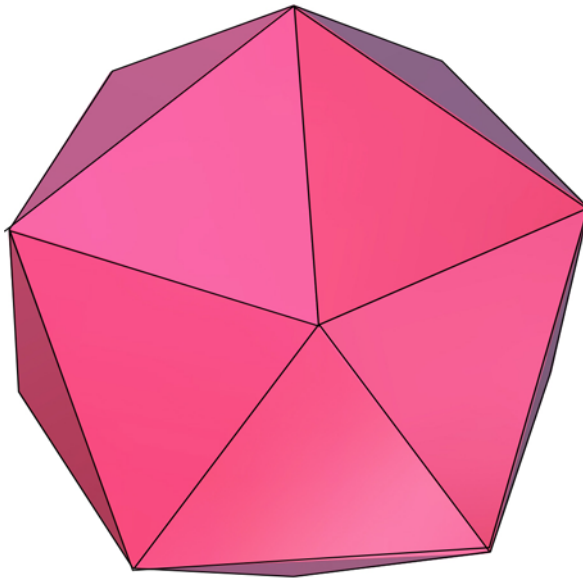
Now place five more triangles around the rim.



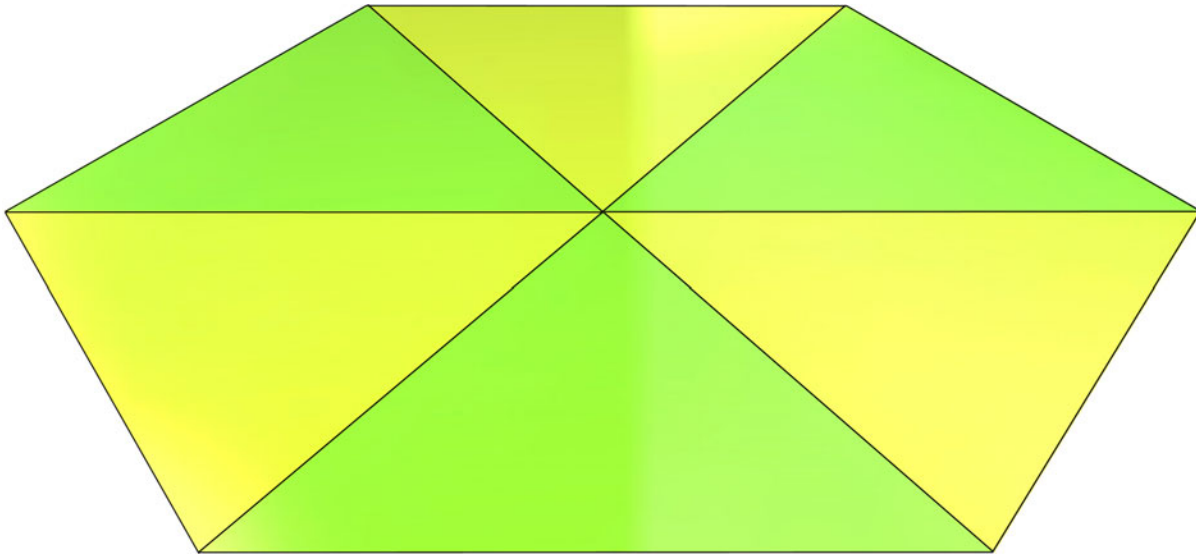
Now start the whole process over again with five more triangles on the plane point-to-point. Swing the new triangles up out of the plane, and place five more triangles around the rim.



Place the second set of ten triangles on top of the first set.



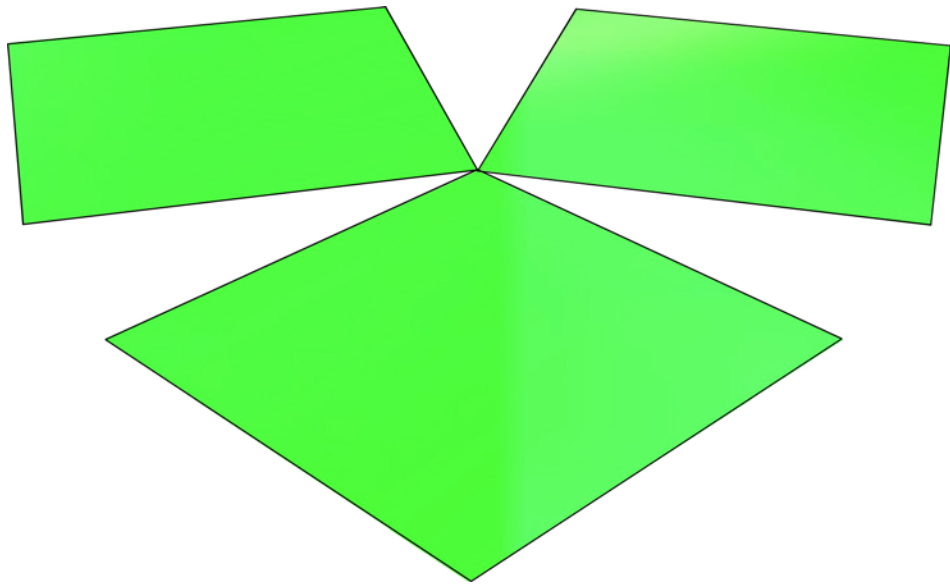
This is the icosahedron, the last of the triangle-based Platonic solids. This shape is the basis for most geodesic domes. The reason why there are no other triangle-based Platonic solids is that, when you place six equilateral triangles together on the plane point-to-point, they are already touching, and so you can't swing them up out of the plane.



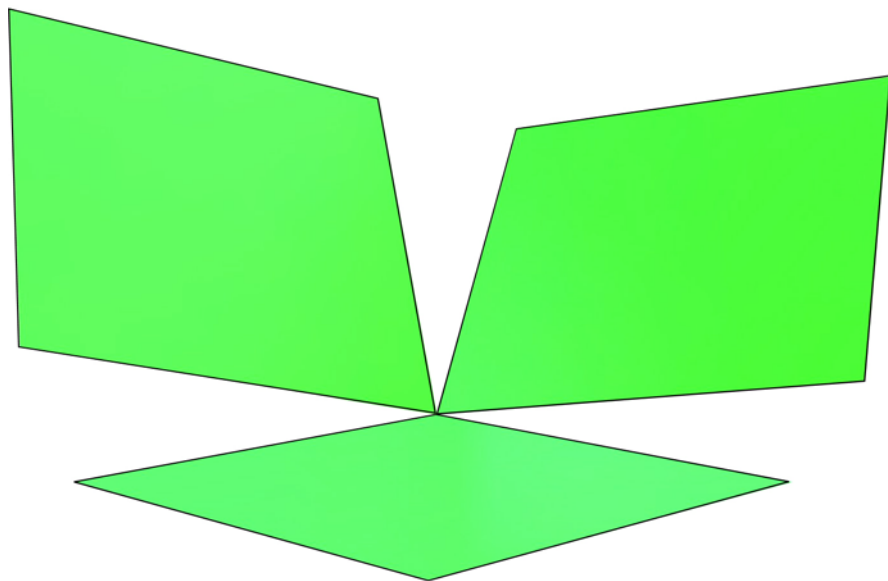
So the only ways that equilateral triangles can meet at the vertices of a platonic solid are in sets of three (the tetrahedron), four (the octahedron), or five (the icosahedron).

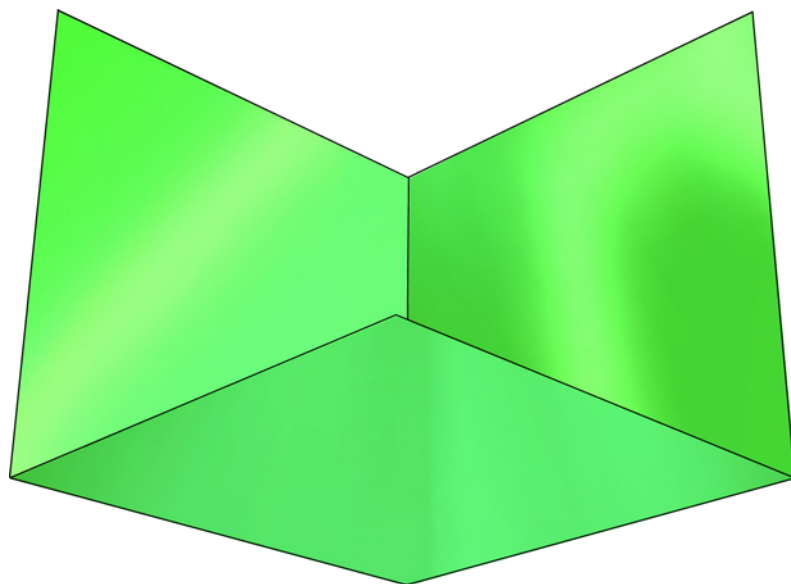
The Cube

I am sure you are familiar with the Platonic solid that is made from squares. It is the familiar cube. It is also known as the hexahedron since “hexa” is Greek for six, and the cube has six faces. To make a cube, place three squares point-to-point on a flat plane.

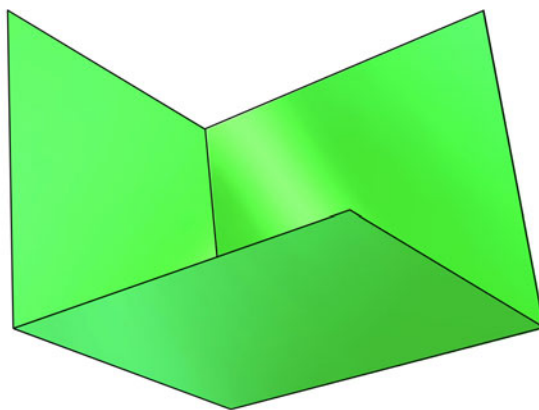
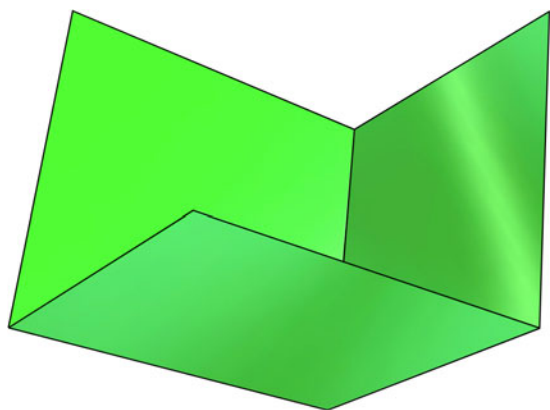


With the center points still on the plane, swing the squares up out of the plane.

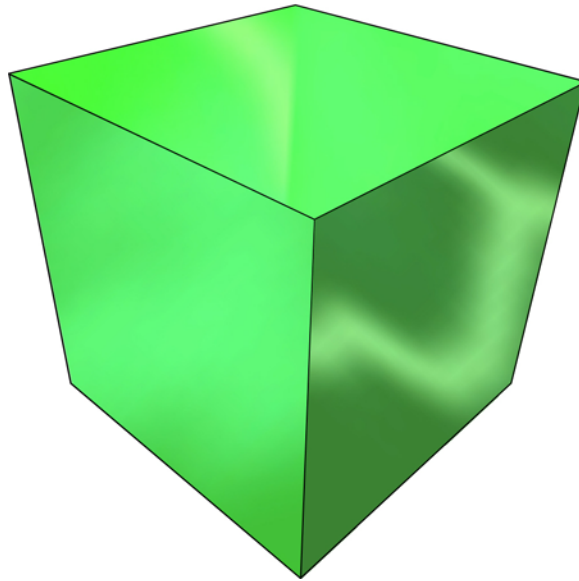




Start the whole process over again with three more squares on the plane point-to-point. Swing the new squares up out of the plane.

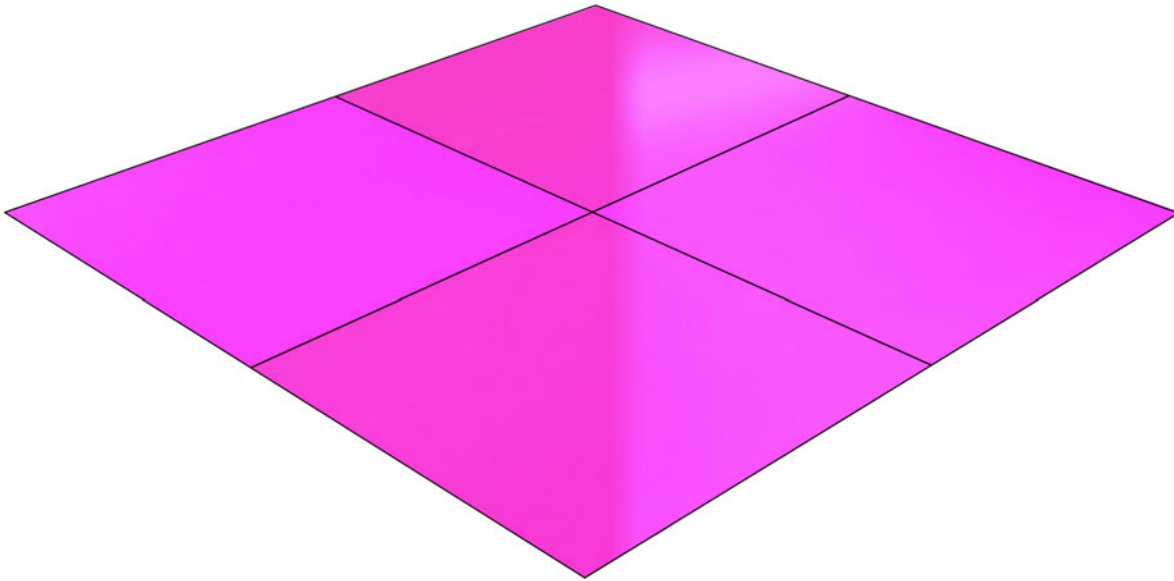


Place the second set of three squares on top of the first set.



This is the cube or hexahedron, the only square-based Platonic solid. Perhaps you do not think the cube fits in with the other Platonic solids. It may seem too ordinary and perhaps not crystalline. Actually, there are several cube-shaped crystals. You will learn in the chapter on dual pairs that the cube fits in perfectly with the other Platonic solids.

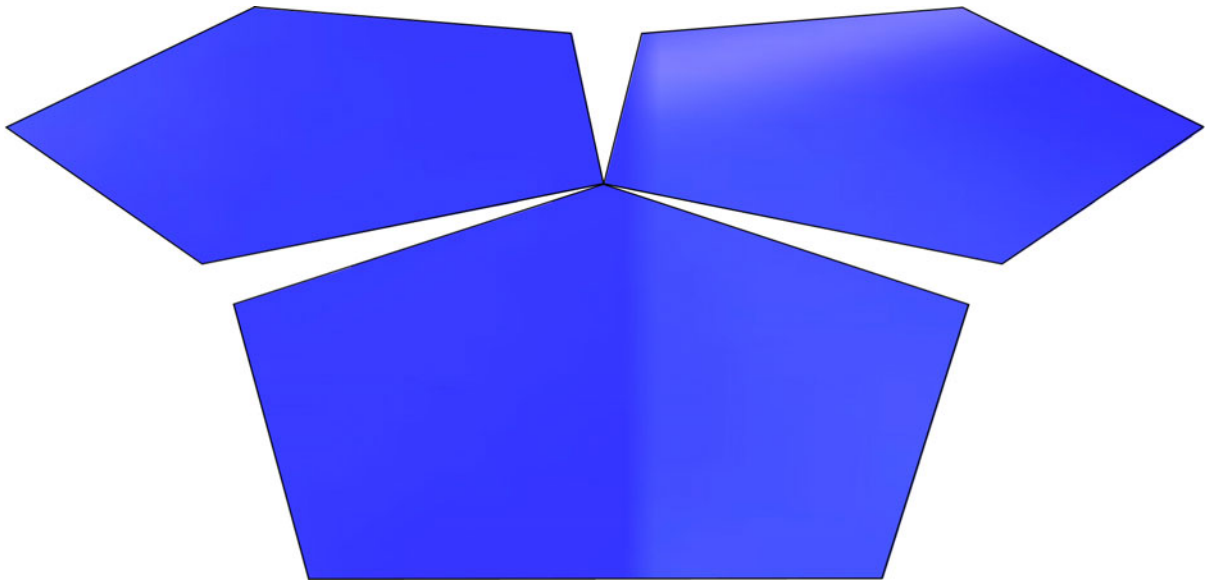
The reason why there are no other square-based Platonic solids is that, when you place four squares together on the plane point-to-point, they are already touching, and so you can't swing them up out of the plane.



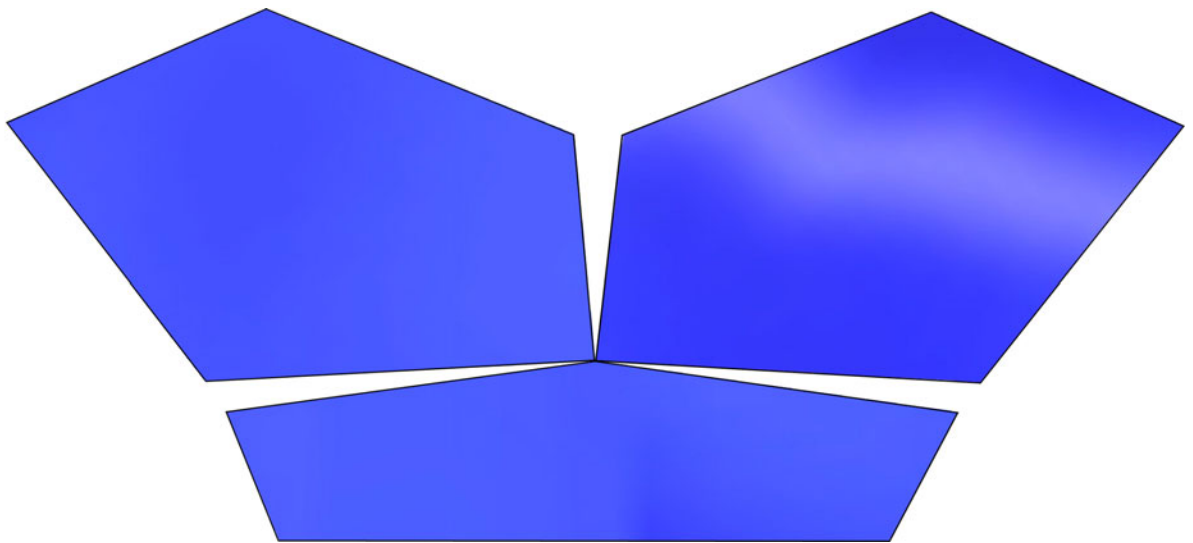
So the only way that squares can meet at the vertices of a platonic solid is in sets of three.

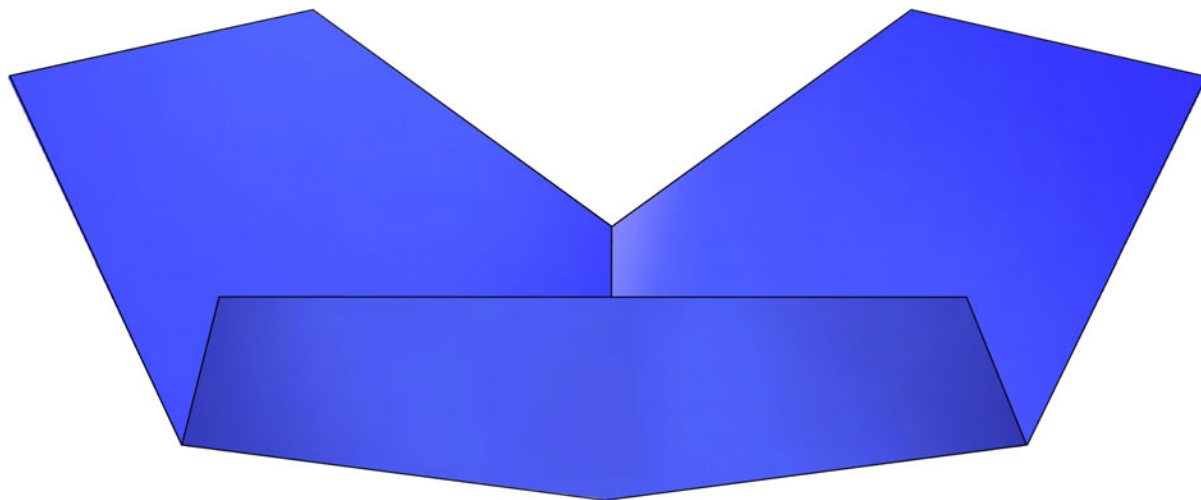
The Dodecahedron

The fifth and last Platonic solid is the dodecahedron. The word “dodeca” is Greek for twelve. The dodecahedron has twelve pentagon faces. To make a dodecahedron, place three pentagons point-to-point on a flat plane.

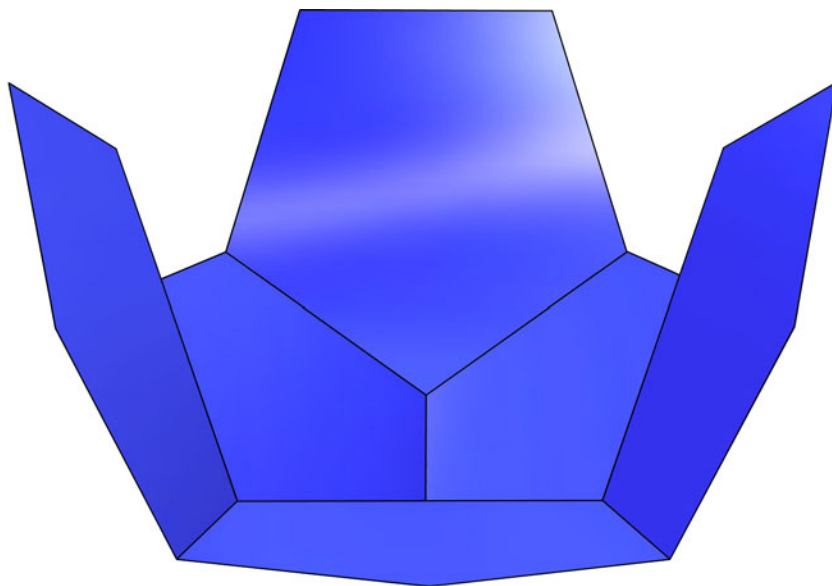


With the center points still on the plane, swing the pentagons up out of the plane.

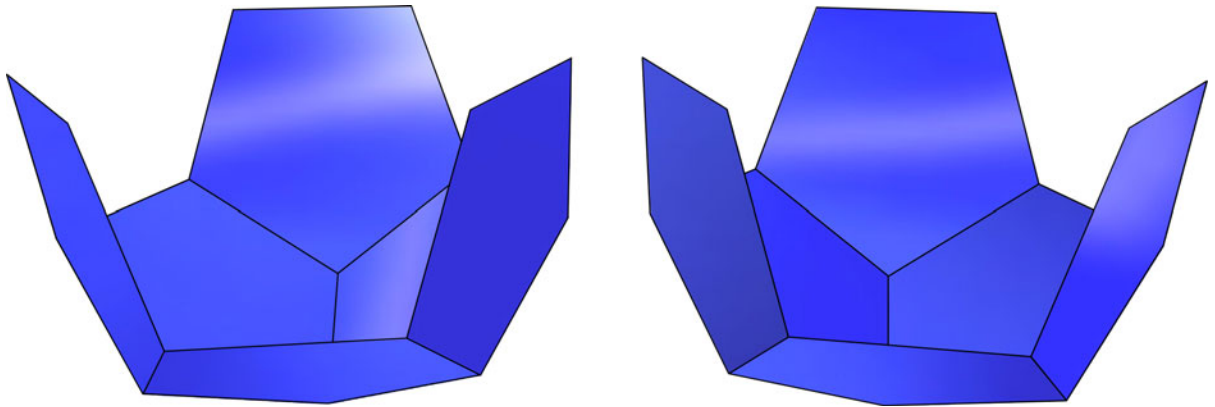




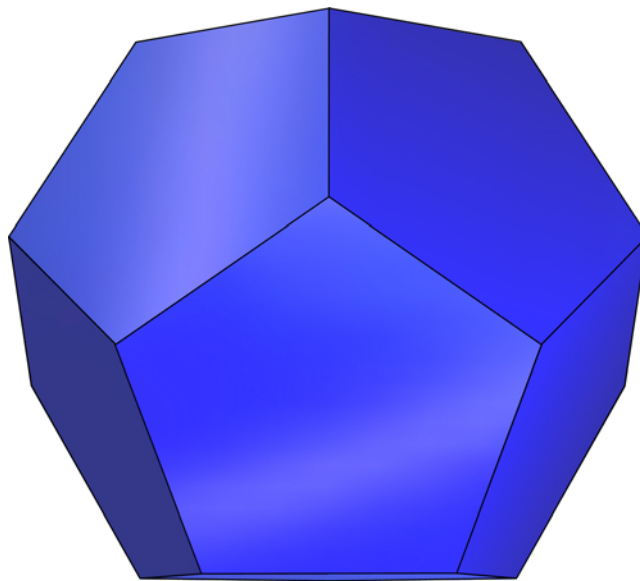
Place three more pentagons around the rim.



Start the whole process over again with three more pentagons on the plane point-to-point. Swing the new pentagons up out of the plane and place three more pentagons around the rim.

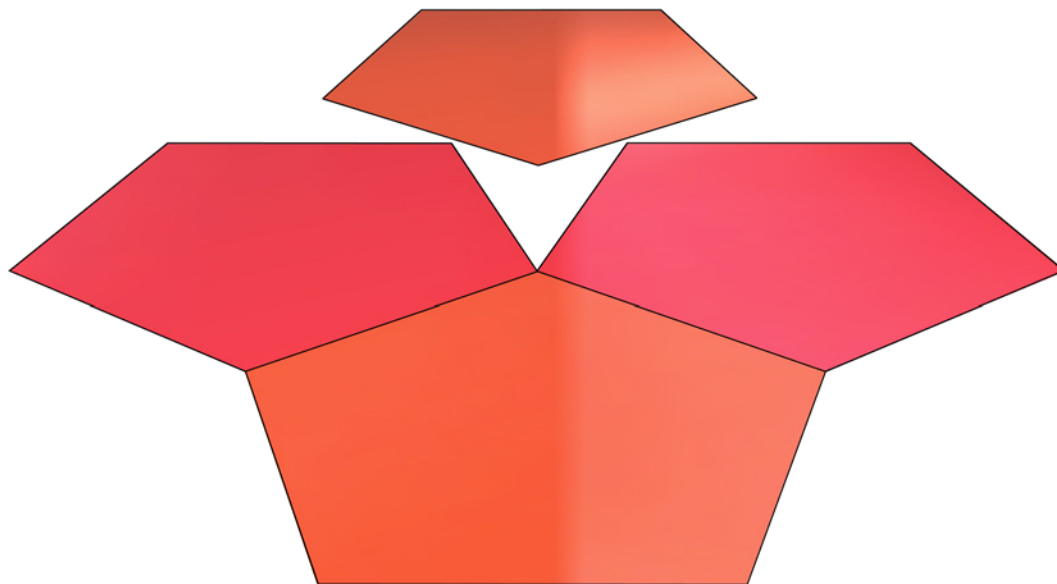


Place the second set of six pentagons on top of the first set.



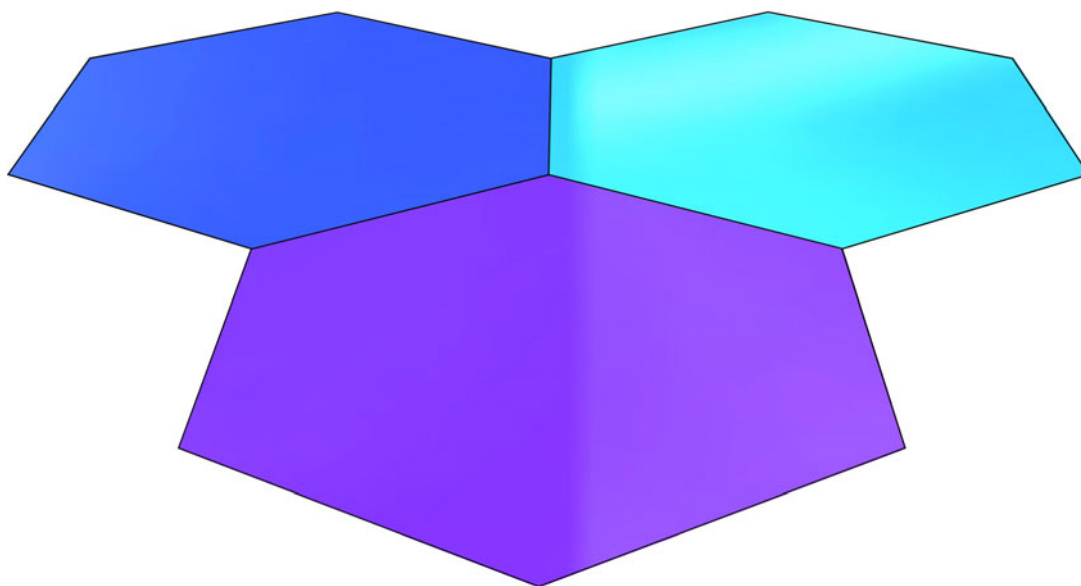
The dodecahedron is the basis of the pattern used for most soccer balls and volley balls.

The reason why there are no other pentagon-based Platonic solids is that you can't place more than three pentagons together on the plane point-to-point. They won't fit together.

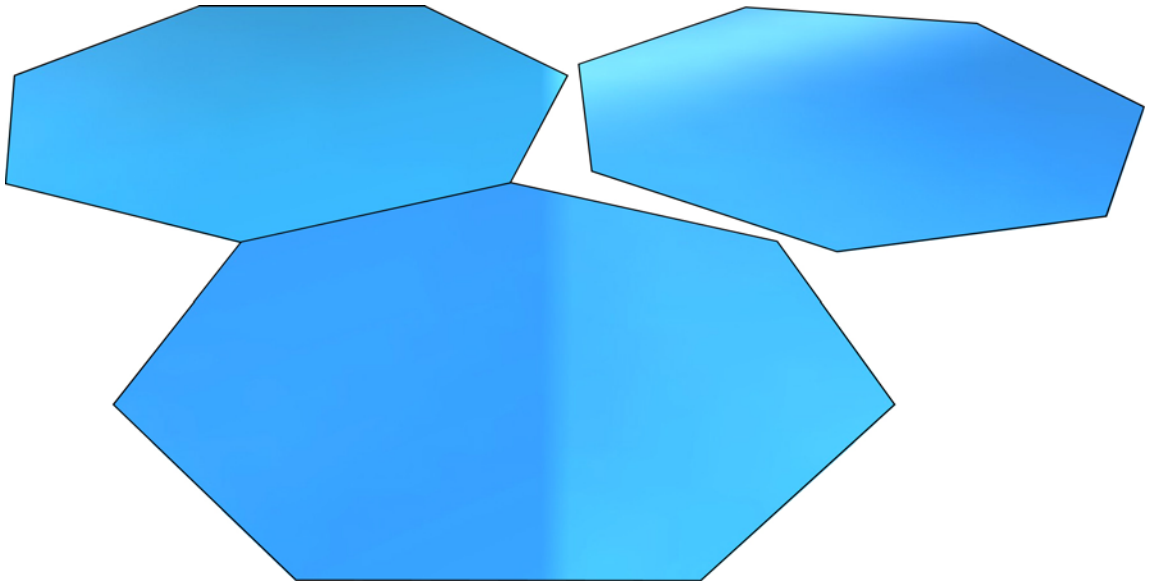


So the only way that pentagons can meet at the vertices of a Platonic solid is in sets of three.

Three hexagons fit together on the plane point-to-point with no room left to swing up.



Three heptagons (seven-sided polygons) won't fit together on the plane point-to-point.



So the dodecahedron is the last of the Platonic solids. For some strange (some might say magical) reason, there are exactly five solids consisting of regular polygons, all of the same type, all fitting together in the same way. These are Plato's five sacred solids.

Questions to Ponder

- ◆ How could the preceding constructions be taught starting each solid with one regular polygon on the plane surrounded by other polygons placed together edge-to-edge?
- ◆ Why is it that, no matter what type and number of identical regular polygons you start with, if they can swing up out of the plane, they always fit together perfectly to complete a closed polyhedron?
- ◆ Why do crystals naturally form in the shapes of Platonic solids?



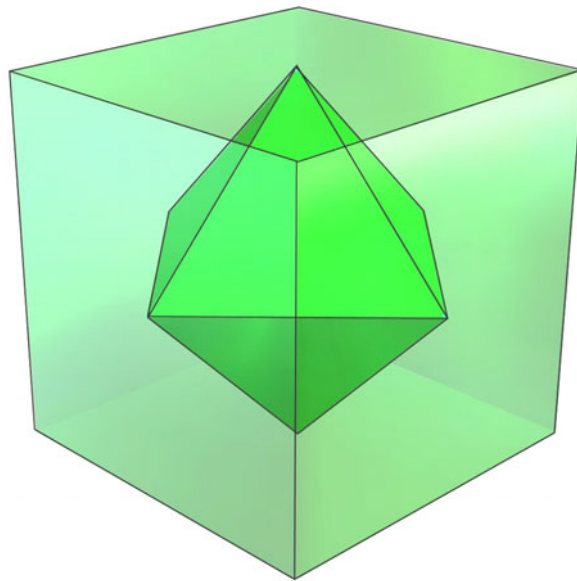
- ◆ Why are people attracted to these forms even when they do not know about the mathematics behind them?
- ◆ How is it that our convention of there being 360 degrees in a full circle results in the fact that the first several regular polygons have integer degree measures in their angles, and how do these angles relate to the fact that there are exactly five Platonic solids?
- ◆ Given the number and type of polygons originally placed point-to-point on the plane, is there a way to calculate the total number of faces that will be in the completed Platonic solid?
- ◆ Is there a way to calculate the dihedral angle (the angle between two faces of the completed Platonic solid)?
- ◆ If one or more of the restrictions on Platonic solids is selectively removed or changed, what other classes of solids result?

CHAPTER 2

DUAL PAIRS



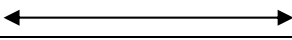
The Cube-Octa Pair

While the cube may not, at first, seem like a Platonic solid, it turns out that there is an octahedron hidden inside of every cube. If you connect the centers of the six faces of a cube, you will discover that they correspond to the six vertices of an octahedron in the interior of the cube.

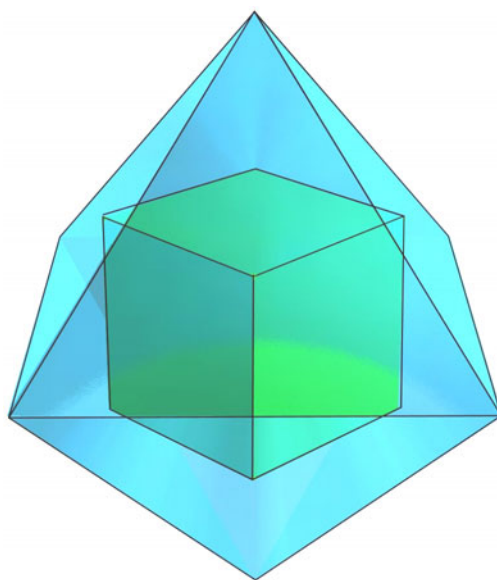


This relationship has to do with the numbers three, four, six, eight, and twelve. The cube has three faces meeting at each vertex, four sides to each face, six faces in all, and eight vertices in all. The octahedron has three sides to each face, four faces meeting at each vertex, six vertices in all, and eight faces in all. In this way, the faces of the cube are analogous to the vertices of the octahedron. Notice also, that they

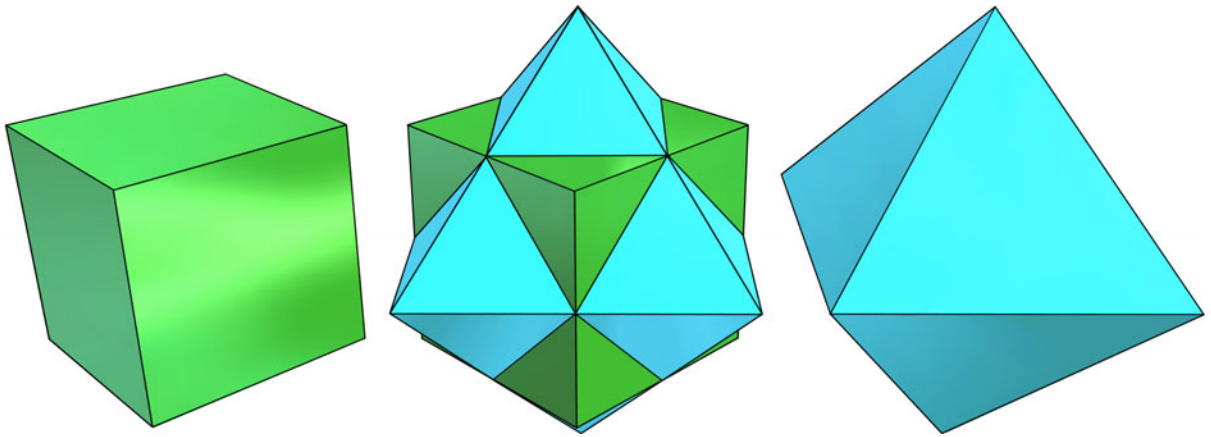
both have exactly twelve edges. The cube and the octahedron are a numerically bound pair.

| cube | | octahedron |
|-----------------------------|-----------------------------------------------------------------------------------|-----------------------------|
| 3 faces meet at each vertex |  | 4 faces meet at each vertex |
| 4 sides on each face | | 3 sides on each face |
| 6 faces in all |  | 8 faces in all |
| 8 vertices in all | | 6 vertices in all |
| 12 edges in all |  | 12 edges in all |

If you connect the centers of the eight faces of an octahedron, you will discover that they correspond to the eight vertices of a cube in the interior of the octahedron.



If you superimpose a cube with an octahedron of the same size, you get a beautiful compound solid.



Notice that each square face has a square-based pyramid protruding from it, and that each triangle face has a triangle-based pyramid protruding from it. You can use the patterns in chapter three to build your own paper models of this and other compound solids.

The Dodeca-Icosa Pair

If you connect the centers of the twelve faces of a dodecahedron, you will discover that they correspond to the twelve vertices of an icosahedron in the interior of the dodecahedron.

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